Let $a, b \in \mathbb{R}^n$ and $p > 1$. Note that

$$\begin{aligned}(|a|^{p-2}a - |b|^{p-2}b)(a-b) &= \int_0^1 |b + s(a-b)|^{p-2}|a-b|^2\,\mathrm{d}s \\ &+ (p-2)\int_0^1 |b+s(a-b)|^{p-4}((b+s(a-b))(a-b))^2\,\mathrm{d}s \\ &\geq |a-b|^2 \int_0^1 |b+s(a-b)|^{p-2}\,\mathrm{d}s.\end{aligned}$$

We set

$$I(a,b) = (|a|^{p-2}a - |b|^{p-2}b)(a-b).$$

Case 1: $1 < p < 2$. For any $0 < s < 1$ we have

$$|sa + (1-s)b|^{p-2} \geq (|a|+|b|)^{p-2},$$

and therefore

$$\text{(A)} \qquad I(a,b) \geq (|a|+|b|)^{p-2}|a-b|^2.$$

Case 2: $p \geq 2$.

Subcase (i): $|a| \geq |a-b|$ and $|b| \geq |a-b|$. Note that if $1/2 < s < 1$, then

$$\begin{aligned}|sa+(1-s)b| &= |a + (1-s)(b-a)| \geq |a| - (1-s)|a-b| \\ &\geq |a-b| - (1-s)|a-b) = s|a-b| \geq \frac{|a-b|}{2},\end{aligned}$$

and if $0 < s < 1/2$, then

$$|sa+(1-s)b| \geq (1-s)|a-b| \geq \frac{|a-b|}{2}.$$

Hence, we get

$$\int_0^1 |sa+(1-s)b|^{p-2}\,\mathrm{d}s \geq 2^{2-p}|a-b|^{p-2},$$

and

$$I(a,b) \geq 2^{2-p}|a-b|^p.$$

Subcase (ii): $|a| < |a-b|$ or $|b| < |a-b|$. We consider only the case where $|a| < |a-b|$. Observe that for $0 < s < 1$,

$$\begin{aligned}|sa+(1-s)b| &= |a + (1-s)(b-a| \leq |a| + (1-s)|a-b| \\ &< (2-s)|a-b| \leq 2|a-b|.\end{aligned}$$

Hence,

$$I(a,b) \geq \frac{1}{4}\int_0^1 |sa+(1-s)b|^p \, \mathrm{d}s.$$

By convexity, we get

$$\left(\int_0^1 |sa+(1-s)b|^2 \, \mathrm{d}s\right)^{p/2} \leq \int_0^1 |sa+(1-s)b|^p \, \mathrm{d}s.$$

Observe that

$$\begin{aligned}\int_0^1 |sa+(1-s)b|^2 \, \mathrm{d}s &= \int_0^1 \big(s^2|a|^2+(1-s)^2|b|^2+2s(1-s)ab\big) \, \mathrm{d}s \\ &= \frac{1}{3}\big(|a|^2+|b|^2+ab\big) \\ &= \frac{1}{3}\frac{|a-b|^2+3|a+b|^2}{4} \geq \left(\frac{|a-b|}{4}\right)^2.\end{aligned}$$

combining the above, we obtain

$$I(a,b) \geq \frac{|a-b|^p}{4^{p+1}}.$$

Thus, if $p \geq 2$, we have

$$\text{(B)} \qquad I(a,b) \geq \left(2^{2-p} \wedge 4^{-p-1}\right)|a-b|^p = 4^{-p-1}|a-b|^p.$$